Computation of the Equivalent Capacitance of a Via in a Multilayered Board Using the Closed-Form Green's Function

Kyung S. Oh, Jose E. Schutt-Aine, Raj Mittra, and Bu Wang

***Abstract*— A method based on the quasi-static approximation for computing the equivalent capacitance of a via is presented in this paper. The geometry of a via consists of traces, pads and a perfectly conducting cylindrical rod; the via is buried in a multilayered dielectric medium with optional reference (ground) planes. The total number of traces, pads, and ground planes can be arbitrary, as well as the angles and cross sections. The method is based on the excess charge formulation of an integral equation applied in conjunction with the recently developed closed-form Green's function.**

I. Introduction

Although a via is one of the most common discontinuities encountered in high-speed integrated circuits, it has not received as much attention as some of the other discontinuities, e.g., open-end terminations, bends, and junctions. This is due mainly to the nonplanar and complex three-dimensional (3-D) geometry of the via, which has often been simplified in the works published previously [1]–[3]. For instance, a via penetrating through a single reference (ground) plane with two wire traces has been considered in [1], and without any traces in [2], while a via above a reference plane with two wire traces but without a through-hole reference plane between these traces has been investigated in [3]. A novel equivalent network model, which accounts for the frequency dependence, has been proposed in [4] and has also been applied to the problem of coupling between two adjacent vias in [5]. In [1] and [3], an integral equation has been formulated in terms of the excess charge distribution to compute the equivalent (excess) capacitance. In this paper, this excess charge formulation is further generalized for vias with more complex geometries than has been analyzed hitherto and is applied in conjunction with the closed-form Green's function to analyze vias embedded in multilayered dielectric media.

A closed-form Green's function for a multilayered dielectric medium was first introduced in [6]. This Green's function utilizes a finite number of complex images and avoids the evaluation of a nested infinite series expression required in the computation of the exact Green's function for a layered medium. In [7], a closed-form Green's function based on weighted real images was proposed and was used to compute the equivalent circuit of a strip crossover. In [8], this closed-form Green's function based on weighted real images was further generalized to handle a semi-infinite line and then applied to compute various strip junction discontinuities. In this paper, the closed-form Green's function discussed in [8] is employed to compute the equivalent circuit for a via in a multilayered dielectric medium.

II. General Statement of the Problem

To illustrate the geometries of vias considered in this paper, a via comprised of three traces and one reference ground plane is shown in Fig. 1(a). In general, a via can pass through N_g reference (ground) planes and N_t traces, and N_p pads can be attached to the via where N_g, N_t, and N_p are all arbitrary. The vias are embedded in a multilayered medium consisting of N_d (arbitrary) dielectric layers, which can be backed by two optional reference planes as shown in Fig. 1(b). To distinguish between these optional reference planes and those associated with the via, we will reserve the term "reference plane" to designate an optional top or bottom reference plane, and use the term "reference conductor" to denote other reference planes. It is evident that the reference conductors must have perforations to avoid any contact with the vias; however, the two optional reference planes are assumed to be solid. To simplify the numerical computation, we assume that all of the conductors are infinitely thin and that the shapes of all the pads and perforations in the reference conductors are circular.

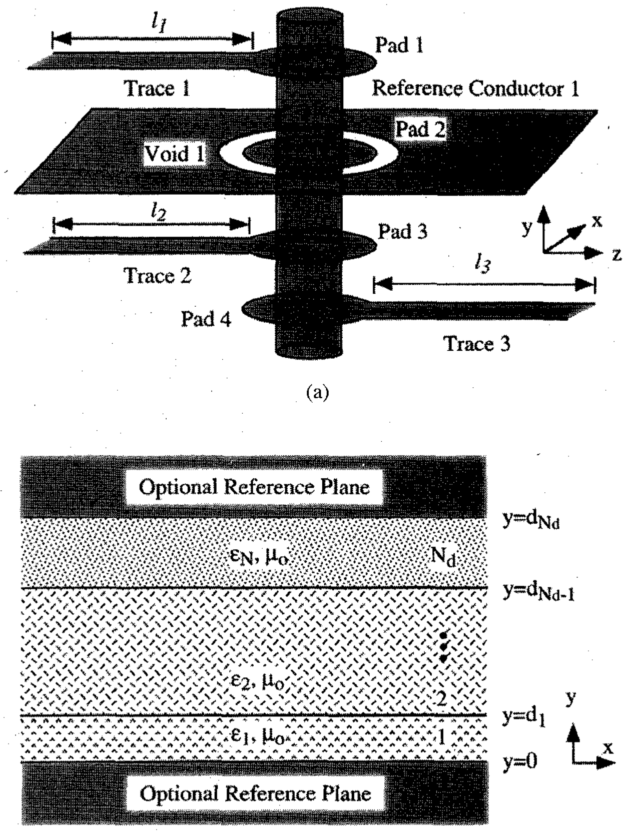

Fig. 1. (a) The geometry of a via, and (b) its surrounding multilayered medium.

A quasi-static equivalent circuit representation of the via shown in Fig. 1 is given in Fig. 2. This paper will only address the problem of computing the total equivalent capacitance C_e. The method to compute the equivalent inductance of a via can be found in [9].

In Section III, an integral equation is formulated in terms of the excess charge distribution using the closed-form Green's function, and the method of moments (MoM) is subsequently employed to determine the unknown charge distribution. A detail discussion of the closed-form Green's function and the corresponding expression can be found in [8]. In Section IV, several numerical examples are presented to verify the proposed method.

III. Formulation of an Integral Equation

An impressed potential on conductors results in free charge accumulation on the surfaces of conductors, and the electrostatic potential

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TABLE I
EQUIVALENT CAPACITANCES FOR VIAS SHOWN IN FIG 3 UNITS ARE IN pF

	Fig 3(a)		Fig 3(b)	
	$\varepsilon_1=\varepsilon_2=\varepsilon_o$	$\varepsilon_1=4\varepsilon_o$, $\varepsilon_2=\varepsilon_o$	$\varepsilon_1=\varepsilon_2=4\varepsilon_o$	$\varepsilon_1=4.5\varepsilon_o$, $\varepsilon_2=5.4\varepsilon_o$
Computed Result	0.3841	1.233	9.952	12 31
Others	0 3701 [3]	1 28 [3]	6 35 [1]	7 85 [1]

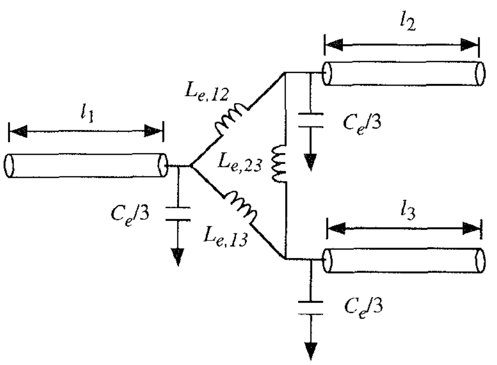

Fig. 2. A equivalent circuit representation of the via shown in Fig. 1(a).

$\phi(r)$ at any point except inside conductors is then related to this surface charge density $q(r)$ via the following integral equation

$$\phi(r) = \int_{\Omega} G^{3D}(r \mid r')q(r')dr' = \langle G^{3D}, q\rangle \quad (1)$$

where Ω denotes the surfaces of all conductors, including the via and reference conductors. $G^{3D}(r \mid r')$ is the 3-D closed-form Green's function for a multilayered medium, and it accounts for polarization charges on dielectric interfaces and free charges on the surfaces of reference planes [8]. The integration is symbolically written as $\langle\cdot,\cdot\rangle$ to simplify the notation. Next, the charge distribution over the whole structure analyzed is decomposed into $q_v(r), q_p^i(r), q_t^i(r)$, and $q_g^i(r)$ where $q_v(r)$ is the charge density on the surface of a via hole, and $q_p^i(r), q_t^i(r)$, and $q_g^i(r)$ are the charge densities on the ith pad, trace, and reference conductor, respectively. Equation (1) can be rewritten as

$$\phi(r) = \langle G^{3D}, q_v\rangle + \sum_{i=1}^{N_p}\langle G^{3D}, q_p^i\rangle + \sum_{i=1}^{N_t}\langle G^{3D}, q_t^i\rangle + \sum_{i=1}^{N_g}\langle G^{3D}, q_g^i\rangle. \quad (2)$$

Next, the charge density $q_t^i(r)$ is further decomposed into the uniform charge density $q_t^{\text{unif},i}(r)$ and the excess charge density $q_t^{\text{excess},i}(r)$

$$q_t^i(r) = q_t^{\text{unif},i}(r) + q_t^{\text{excess},i}(r). \quad (3)$$

Here, $q_t^{\text{unif},i}(r)$ is the uniform charge density on the ith trace under the assumptions that it is infinite in both directions, no other traces are present, and the reference conductors have no perforations; this charge density is computed by solving an appropriate two-dimensional (2-D) problem. Since the reference conductors become uniform planes without any perforations for this 2-D problem, the potential distribution on the region above the reference conductor is not affected by the region below it and vice versa. To solve for $q_t^{\text{unif},i}(r)$, it is then expedient to introduce a new medium surrounding the ith trace. As a consequence, the medium employed in the 2-D problem is generally different from that of the 3-D via problem, and could, in fact, be different for each trace. Once the appropriate medium has been chosen, $q_t^{\text{unif},i}(r)$ can be obtained by using the method described in [7]. The resulting $q_t^{\text{unif},i}(r)$ yields the capacitance per unit length for the ith transmission line in the equivalent circuit representation shown in Fig. 2.

In the process of determining $q_t^{\text{unif},i}(r)$, the 2-D closed-form Green's function $G^{2D,i}(\rho \mid \rho_0)$ is used to formulate an integral equation for a 2-D problem [7]. However, in the integral (2) for computing the equivalent capacitance problem, the uniform charge density $q_t^{\text{unif},i}(r)$ resides on the ith trace, which is only a semi-infinite line. It is therefore necessary to employ $G^{\text{semi},i}(r \mid r_0, \xi)$ to compute the potential due to $q_t^{\text{unif},i}(r)$ [8]. Using (3), (2) can be rewritten as

$$\phi(r) - \sum_{i=1}^{N_t}\langle G^{\text{semi},i}, q_t^{\text{unif},i}\rangle = \langle G^{3D}, q_v\rangle + \sum_{i=1}^{N_p}\langle G^{3D}, q_p^i\rangle + \sum_{i=1}^{N_t}\langle G^{3D}, q_t^{\text{excess},i}\rangle + \sum_{i=1}^{N_g}\langle G^{3D}, q_g^i\rangle. \quad (4)$$

If we set the via potential to be ϕ_0 with respect to the reference conductors and planes, $\phi(r)$ becomes ϕ_0 on the surfaces of the via hole, pads, and traces, and is equal to 0 on the surfaces of reference conductors. Hence, once $q_t^{\text{unif},i}(r)$ has been determined, all of the quantities associated with the left-hand side of (4) can be considered to be known at the surface of the conductors, and the method of moments can be applied to solve (4). The various integrations appearing in (4) can be evaluated analytically for pulse-type basis functions using closed-form formulas given in [8].

Once the unknown charge distributions have been determined, the equivalent (excess) capacitance C_e can be obtained by using the following expression which involves the integrals of these charge distributions

$$C_e\phi_0 = \int_{\Omega_v} q_v(r')dr' + \sum_{i=1}^{N_p}\int_{\Omega_{p,i}} q_p^i(r')dr' + \sum_{i=1}^{N_t}\int_{\Omega_{t,i}} q_t^{\text{excess},i}(r')dr' + \sum_{i=1}^{N_g}\int_{\Omega_{g,i}} q_g^i(r')dr' \quad (5)$$

where Ω_v is the surface of a via hole, $\Omega_{p,i}$, $\Omega_{t,i}$, and $\Omega_{g,i}$ and are the surfaces of the ith pad, trace, and reference conductor, respectively.

IV. NUMERICAL EXAMPLES

We will now present two numerical examples to illustrate the application of the method presented above to the computation of the equivalent capacitances of two via structures, one with a reference plane and the other with a reference conductor. The detailed geometries of the two via structures are shown in Fig. 3. The computed

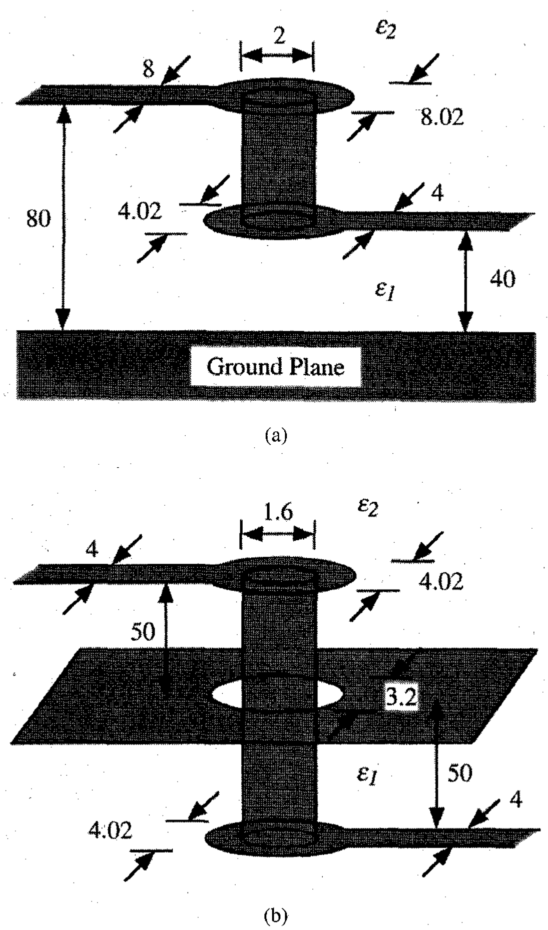

Fig. 3. (a) A two-trace via with a reference plane and (b) a two-trace via with a reference conductor. All dimensions are in mm.

excess capacitances for Fig. 3(a) with 1) $\varepsilon_1 = \varepsilon_2 = \varepsilon_0$ and 2) $\varepsilon_1 = 4\varepsilon_0$ and $\varepsilon_2 = \varepsilon_0$; for Fig. 3(b) with 3) $\varepsilon_1 = \varepsilon_2 = 4\varepsilon_0$ and 4) $\varepsilon_1 = 4.5\varepsilon_0$ and $\varepsilon_2 = 5.4\varepsilon_0$ are listed in Table I along with data obtained from [1] and [3]. In [1] and [3], the strips were replaced by the equivalent wires of radii which are one-fourth of the widths of the strips. In our computation, the lengths of all traces have been truncated to $2.5h$, whereas the width of the reference conductor has been truncated to $1.5l$, with h and l being the height of a via hole and the length of the traces. The truncation of traces and reference conductors is valid since the excess charge distribution decays rapidly as we move away from the center of a via. A total of 263 and 687 unknowns were used for the vias shown in Figs. 3(a) and (b), respectively. As shown in Table I, the data for the via shown in Fig. 3(a) agree well with the published results. However, the data for the via shown in Fig. 3(b) are considerably different from the results reported elsewhere. Unfortunately, no experimental result for this structure is available to establish the relative accuracy of these results associated with Fig. 3(b).

V. Conclusion

A method to compute the equivalent capacitance of a via, which is based on an integral equation formulated in terms of the excess charge formulation, has been presented in this paper. The method is applicable to via geometries with or without through-hole reference conductors. The recently developed closed-form Green's function was employed to circumvent the time-consuming evaluation of a nested infinite series, required in the evaluation [3].

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Analysis of Edge Coupled Strip Inset Dielectric Guide

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***Abstract*—The edge coupled strip inset dielectric guide is analyzed using the extended spectral domain approach. This structure, as compared to microstrip line, has several interesting features and can be very useful for microwave and millimeter wave applications. Validity of the approach is established by comparing numerical results with measured data. As many structural and material parameters can be chosen, a wide fundamental mode bandwidth and a broad range of characteristic impedances can be achieved, leading to great flexibility. The dispersion in fundamental mode propagation constants and impedances is found to be very low. With suitable choice of different permittivities for two dielectric layers, the same propagation constants for two fundamental modes can be obtained. This property is desirable for directional coupler applications.**

I. Introduction

Microstrip line has been the most popular transmission medium used for constructing microwave and millimeter wave circuits [1]. It is well known that one of the problems with open microstrip circuits is the excitation of surface waves from discontinuities in the circuits

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